The branching ratio R_b in the littlest Higgs model

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Abstract

In the context of the littlest Higgs(LH) model, we study the contributions of the new particles to the branching ratio R_b. We find that the contributions mainly dependent on the free parameters f, c' and x_L. The precision measurement value of R_b gives severe constraints on these free parameters.

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I. Introduction

It is well know that most of the electroweak oblique and QCD corrections to the $Z \to b\bar{b}$ branching ratio R_b cancel between numerator and denominator, R_b is very sensitive to the new physics(NP) beyond the standard model(SM). The precision experimental value of R_b may give a severe constraint on the NP[1]. Thus, it is very important to study the $Z \to b\bar{b}$ process in extensions of the SM and pursue the resulting implications.

Little Higgs models[2,3,4] provide a new approach to solve the hierarchy between the TeV scale of possible NP and the electroweak scale, $\nu = 246GeV = (\sqrt{2}G_F)^{-1/2}$. In these models, at least two interactions are needed to explicitly break all of the global symmetries, which forbid quadratic divergences in the Higgs mass at one-loop. Electroweak symmetry breaking(EWSB) is triggered by a Coleman-Weinberg potential, which is generated by integrating out the heavy degrees of freedom. In this kind of models, the Higgs boson is a pseudo-Goldstone boson of a global symmetry which is spontaneously broken at some higher scale f by an vacuum expectation value(VEV) and thus is naturally light. A general feature of this kind of models is that the cancellation of the quadratic divergences is realized between particles of the same statistics.

Little Higgs models are weakly interaction models, which contain extra gauge bosons, new scalars and fermions, apart from the SM particles. These new particles might produce characteristic signatures at the present and future collider experiments[5,6,7]. Since the extra gauge bosons can mix with the SM gauge bosons W and Z, the masses of the SM gauge bosons W and Z and their couplings to the SM particles are modified from those in the SM at the order of $\frac{\nu^2}{f^2}$. Thus, the precision measurement data can give severe constraints on this kind of models[5,8,9].

Aim of this paper is to consider the $Z \to b\bar{b}$ branching ratio R_b in the context of the littlest Higgs(LH) model[2] and see whether the new particles predicted by the LH model can give significant contributions to R_b. We find that, compare the calculated value of R_b with the experimental measured value, the precision data can give severe constraint on the free parameters of the LH model.

The LH model has been extensively described in literature. However, in order to

clarify notation which is relevant to our calculation, we will simply review the LH model in section II. In section III, we discuss the effects of the new gauge bosons on the branching ratio R_b. We calculate the contributions of the top quark t and vector-like quark T to R_b via the couplings $W\bar{t}b, W\overline{T}b, W'\bar{t}b$ and $W'\overline{T}b$ in section IV. The contributions of the new scalars to R_b are studied in section V. Discussions and conclusions are given in section VI.

II. Littlest Higgs model

The LH model[2] is embedded into a non-linear σ model with the coset space of $SU(5)/SO(5)$. At the scale $\Lambda_s \sim 4\pi f$, the global $SU(5)$ symmetry is broken into its subgroup $SO(5)$ via a VEV of order f, resulting in 14 Goldstone bosons. The effective field theory of these Goldstone bosons is parameterized by a non-linear σ model with gauge symmetry $[SU(2) \times U(1)]^2$, spontaneously broken down to its diagonal subgroup $SU(2) \times U(1)$, identified as the SM electroweak gauge group. Four of these Goldstone bosons are eaten by the broken gauge generators, leaving 10 states that transform under the SM gauge group as a doublet H and a triplet Φ. A new charge 2/3 quark T is also needed to cancel the divergences from the top quark loop.

The effective non-linear Lagrangian invariant under the local gauge group $[SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$, which can be written as[5,9]:

$$\pounds_{eff} = \pounds_G + \pounds_F + \pounds_Y + \pounds_\Sigma - V_{CW}, \tag{1}$$

where $\pounds_G$ consists of the pure gauge terms, which can give the 3− and 4−particle interactions among the $SU(2)$ gauge bosons and the couplings of the $U(1)$ gauge bosons to the $SU(2)$ gauge bosons. The fermion kinetic term $\pounds_F$ can give the couplings of the gauge bosons to fermions. The couplings of the scalars H and Φ to fermions can be derived from the Yukawa interaction term $\pounds_Y$. In the LH model, the global symmetry prevents the appearance of a Higgs potential at tree level. The effective Higgs potential, the Coleman-Weinberg potential V_{CW}[10], is generated at one-loop and higher orders due to interactions with gauge bosons and fermions, which can induce to EWSB by driving the Higgs mass squared parameter negative. $\pounds_\Sigma$ consists of the σ model terms of the LH

model. The scalar Σ field can be written as:

$$\Sigma = e^{i\Pi/f}\langle\Sigma_0\rangle e^{i\Pi^T/f} = e^{2i\Pi/f}\langle\Sigma_0\rangle \tag{2}$$

with $\langle\Sigma_0\rangle \sim f$ which generates masses and mixing between the gauge bosons. The ten pseudo-Goldstone bosons can be parameterized as:

$$\Pi = \begin{pmatrix} 0 & H^+/\sqrt{2} & \Phi^+ \\ H/\sqrt{2} & 0 & H^*/\sqrt{2} \\ \Phi & H^T/\sqrt{2} & 0 \end{pmatrix}. \tag{3}$$

Where H is identified as the SM Higgs doublet, $H = (H^+, H^0)$, and Φ is a complex $SU(2)$ triplet with hypercharge $Y = 2$,

$$\Phi = \begin{pmatrix} \Phi^{++} & \Phi^+/\sqrt{2} \\ \Phi^+/\sqrt{2} & \Phi^0 \end{pmatrix}. \tag{4}$$

The kinetic terms of the scalar Σ field are given by

$$\pounds_\Sigma = \frac{f^2}{8} Tr\{(D_\mu\Sigma)(D^\mu\Sigma)^+\}, \tag{5}$$

where the covariant derivative of the Σ field is defined as:

$$D_\mu\Sigma = \partial_\mu\Sigma - i\sum_j [g_j W_j^a (Q_j^a\Sigma + \Sigma Q_j^{aT}) + g'_j B_j (Y_j\Sigma + \Sigma Y_j^T)], \tag{6}$$

where g_j, g'_j are the gauge coupling constants, W_j, B_j are the gauge bosons, Q_j^a and Y_j are the generators of gauge transformations. The gauge boson mass eigen-states are given by

$$\begin{aligned} W &= sW_1 + cW_2, \quad W' = -cW_1 + sW_2, \\ B &= s'B_1 + c'B_2, \quad B' = -c'B_1 + s'B_2 \end{aligned} \tag{7}$$

with the cosines of two mixing angles,

$$c = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \qquad c' = \frac{g'_1}{\sqrt{g_1'^2 + g_2'^2}}. \tag{8}$$

The SM gauge coupling constants are $g = g_1 s = g_2 c$ and $g' = g'_1 s' = g'_2 c'$.

From the effective non-linear Lagrangian $\mathcal{L}$, one can derive the mass and coupling expressions of the gauge bosons, scalars and the fermions, which have been extensively discussed in Refs.[5,9]. The mass spectrum of the LH model and the coupling forms which are related our calculations are summarized in appendix A , B and C, respectively.

III. New gauge bosons and the $Z \to b\overline{b}$ branching ratio R_b

1. Corrections of new physics to the $Z \to b\overline{b}$ branching ratio R_b

In general, the effective $Z \to b\overline{b}$ vertex can be written as:

$$[g_L^b \overline{b}_L \gamma^\mu b_L + g_R^b \overline{b}_R \gamma^\mu b_R] \cdot Z^\mu \tag{9}$$

with the form factors g_L^b and g_R^b:

$$\begin{aligned} g_L^b &= g_L^{b,SM} + \delta g_L^b = \frac{e}{S_W C_W}(-\frac{1}{2} + \frac{1}{3}S_W^2) + \delta g_L^b, \\ g_R^b &= g_R^{b,SM} + \delta g_R^b = \frac{e}{S_W C_W}(\frac{1}{3}S_W^2) + \delta g_R^b. \end{aligned} \tag{10}$$

Where $S_W = \sin\theta_W$, θ_W is the Weinberg angle. δg_L^b and δg_R^b represent the corrections of NP to the left-handed and right-handed $Zb\overline{b}$ couplings, respectively. Certainly, the corrections of NP to the $Zb\overline{b}$ couplings g_L^b and g_R^b may give rise to one additional form factor, proportional to $\sigma^{\mu\nu}q^\nu$. However, its contributions to R_b are very small and can be ignored[1].

The branching ratio R_b can be written as:

$$R_b = \frac{\Gamma_b}{\Gamma_h} = \frac{\Gamma_b}{3\Gamma_b + 2\Gamma_c}. \tag{11}$$

Here Γ_c is the width of the process $Z \to c\overline{c}$. The partial decay width, Γ_q, of the $Z \to q\overline{q}$ decay($q = u, d, c, s$, and b) is given as[11]:

$$\Gamma_q = 6\Gamma_0(1 + \frac{\alpha_s}{\pi})[(g_L^q)^2 + (g_R^q)^2], \tag{12}$$

with $\Gamma_0 = \frac{G_F m_Z^3}{24\sqrt{2}\pi}$. The factor $\frac{\alpha_s}{\pi}$ contain contributions from the find state gluons and photons. In above equation, the masses of the final quarks are assumed to be negligible.

Since the branching ratio R_b is the ratio between two hadronic widths, R_b is almost independent of the EW oblique and QCD corrections because of the near cancellation of these corrections between the numerator and the denominator. The remaining ones are absorbed in the definition of the renormalized coupling parameters α and S_W, up to terms of high order in the electroweak corrections[12]. Thus, R_b is very sensitive to the NP beyond the SM. The correction of NP to R_b can be written as:

$$\begin{aligned}\delta R_b &= R_b - R_b^{SM} = \frac{\Gamma_b^{SM}+\delta\Gamma_b}{\Gamma_h^{SM}+\delta\Gamma_h} - \frac{\Gamma_b^{SM}}{\Gamma_h^{SM}} \approx (\frac{\delta\Gamma_b}{\Gamma_b} - \frac{\delta\Gamma_h}{\Gamma_h})R_b^{SM} \\ &= R_b^{SM}\{\frac{2(g_L^b\delta g_L^b + g_R^b\delta g_R^b)}{(g_L^b)^2+(g_R^b)^2} - \frac{4(g_L^c\delta g_L^c + g_R^c\delta g_R^c) + 6(g_L^b\delta g_L^b + g_R^b\delta g_R^b)}{2[(g_L^c)^2+(g_R^c)^2]+3[(g_L^b)^2+(g_R^b)^2]}\}. \quad (13)\end{aligned}$$

In above equation, we have neglected the terms of $O[(\delta g_{L,R}^q)^2]$. In the next subsection, we will study the corrections of the new gauge bosons predicted by the LH model to the branching ratio R_b.

2. The extra gauge bosons and the branching ratio R_b

The LH model predicts the existence of the extra gauge bosons, such as W', Z' and B'. These new particles can generate corrections to the branching ratio R_b via mixing with the SM gauge bosons and the coupling to the SM fermions. The corrections to R_b mainly come from three sources: (1) the modifications of the relations between the SM parameters and the precision electroweak input parameters, which come from the mixing of the heavy W' boson to the couplings of the charge current and from the contributions of the current to the equations of motion of the heavy gauge bosons, (2) the correction terms to the $Zb\bar{b}$ couplings g_L^b and g_R^b coming from the mixing between the extra gauge boson Z' and the SM gauge boson Z, (3) the neutral gauge bosons Z' exchange and B' exchange. In the LH model, the relation between the Fermi coupling constant G_F, the gauge boson Z mass m_Z and the fine structure coupling constant $\alpha(m_Z)$ can be written as[9]:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2\sqrt{2}m_Z^2 S_W^2 C_W^2}[1 - \frac{g}{G_F}\frac{c}{s}(c^2-s^2)\frac{\nu^2}{f^2} + 2c^4\frac{\nu^2}{f^2} - \frac{5}{4}(c'^2-s'^2)^2\frac{\nu^2}{f^2}]. \quad (14)$$

So we have

$$\frac{e^2}{S_W^2 C_W^2} = \frac{8G_F m_Z^2}{1 - \frac{g}{G_F}\frac{c}{s}(c^2-s^2)\frac{\nu^2}{f^2} + 2c^4\frac{\nu^2}{f^2} - \frac{5}{4}(c'^2-s'^2)^2\frac{\nu^2}{f^2}}. \quad (15)$$

In our numerical calculations, we will take $G_F = 1.16637\times10^{-5}GeV^{-2}$, $m_Z = 91.187GeV$ and $m_t = 174.3GeV$ [13] as input parameters and use them to represent the other SM parameters.

Due to the mixing between the gauge bosons Z and Z', the tree-level $Zq\overline{q}$ couplings $g_L^{q,SM}$ and $g_R^{q,SM}$ receive corrections at the order of $\frac{\nu^2}{f^2}$:

$$\delta g_L^{q_i,1} = \frac{e}{S_W C_W}\frac{\nu^2}{f^2}[\frac{c^2(c^2-s^2)}{4}+\frac{5}{6}(c'^2-s'^2)(-\frac{1}{5}+\frac{1}{2}c'^2)], \tag{16}$$

$$\delta g_R^{q_i,1} = \frac{e}{S_W C_W}\frac{\nu^2}{f^2}[\frac{5}{3}(c'^2-s'^2)(\frac{1}{5}-\frac{1}{2}c'^2)], \tag{17}$$

$$\delta g_L^{q_j,1} = \frac{e}{S_W C_W}\frac{\nu^2}{f^2}[-\frac{c^2(c^2-s^2)}{4}+\frac{5}{6}(c'^2-s'^2)(\frac{4}{5}-\frac{1}{2}c'^2)], \tag{18}$$

$$\delta g_R^{q_j,1} = \frac{e}{S_W C_W}\frac{\nu^2}{f^2}[\frac{5}{3}(c'^2-s'^2)(\frac{1}{10}+\frac{1}{2}c'^2)], \tag{19}$$

where q_i and q_j represent the down-type quarks(d, s, b) and the up-type quarks(c, s),respectively.

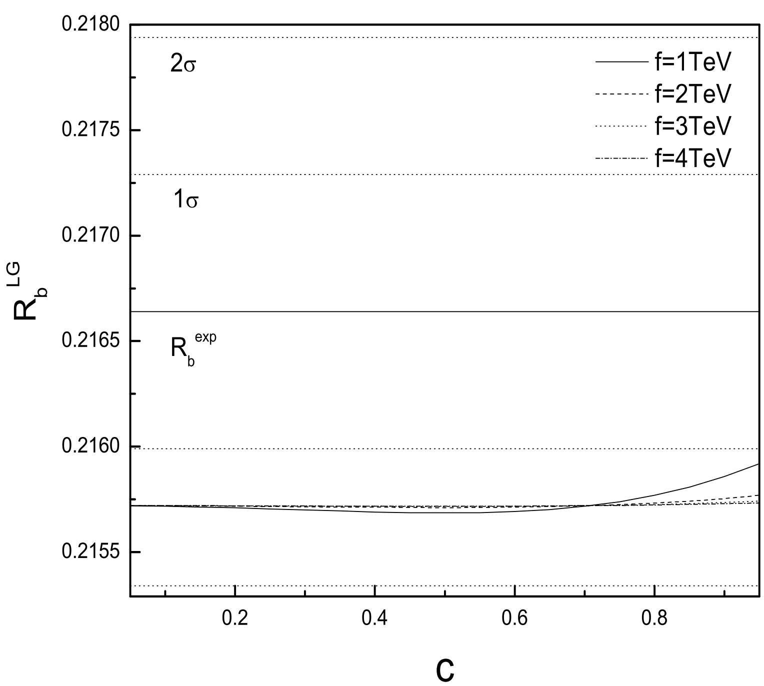

Figure 1: The branching ratio R_b^{LG} as a function of the mixing parameter c for the mixing parameter $c' = \frac{1}{\sqrt{2}}$, $f = 1TeV$, $2TeV$, $3TeV$ and $4TeV$.

Using the same method as calculating the contributions of the topcolor gauge bosons to R_b[14,15], we can give the corrections of the neutral gauge boson Z' exchange and B' exchange to the $Zq\overline{q}$ couplings g_L^q and g_R^q:

$$\delta g_L^{q_i,2} = \frac{e^2c^2}{24\pi^2 S_W^2 s^2}\frac{m_Z^2}{M_{Z'}^2}\ln\frac{M_{Z'}^2}{m_Z^2}g_L^{q_i,SM}, \quad \delta g_R^{q_i,2}=0, \tag{20}$$

$$\delta g_L^{q_j,2} = \frac{e^2c^2}{24\pi^2 S_W^2 s^2}\frac{m_Z^2}{M_{Z'}^2}\ln\frac{M_{Z'}^2}{m_Z^2}g_L^{q_j,SM}, \quad \delta g_R^{q_j,2}=0, \tag{21}$$

$$\delta g_L^{q_i,3} = \frac{e^2}{54\pi^2 C_W^2 s'^2 c'^2}[\frac{1}{5}-\frac{1}{2}c'^2]^2\frac{m_Z^2}{M_{B'}^2}\ln\frac{M_{B'}^2}{m_Z^2}g_R^{q_i,SM}, \tag{22}$$

$$\delta g_R^{q_i,3} = \frac{2e^2}{27\pi^2 C_W^2 s'^2 c'^2}[-\frac{1}{5}+\frac{1}{2}c'^2]^2\frac{m_Z^2}{M_{B'}^2}\ln\frac{M_{B'}^2}{m_Z^2}g_R^{q_i,SM}, \tag{23}$$

$$\delta g_L^{q_j,3} = \frac{e^2}{54\pi^2 C_W^2 s'^2 c'^2}[\frac{1}{5}-\frac{1}{2}c'^2]^2\frac{m_Z^2}{M_{B'}^2}\ln\frac{M_{B'}^2}{m_Z^2}g_R^{q_j,SM}, \tag{24}$$

$$\delta g_R^{q_j,3} = \frac{8e^2}{27\pi^2 C_W^2 s'^2 c'^2}[\frac{1}{5}-\frac{1}{2}c'^2]^2\frac{m_Z^2}{M_{B'}^2}\ln\frac{M_{B'}^2}{m_Z^2}g_R^{q_j,SM}. \tag{25}$$

Where $M_{Z'}$ and $M_{B'}$ are the masses of the gauge boson Z' and the heavy photon B', respectively, which have been listed in appendix A. In above equations, we have used the expressions of the $Z'q\overline{q}$ and $B'q\overline{q}$ couplings given in appendix B. Adding all the corrections together, we obtain the total corrections of extra gauge bosons to R_b:

$$\delta g_L^{b,G} = \delta g_L^{b,1}+\delta g_L^{b,2}+\delta g_L^{b,3}, \qquad \delta g_R^{b,G}=\delta g_R^{b,1}+\delta g_R^{b,3}, \tag{26}$$

$$\delta g_L^{c,G} = \delta g_L^{c,1}+\delta g_L^{c,2}+\delta g_L^{c,3}, \qquad \delta g_R^{c,G}=\delta g_R^{c,1}+\delta g_R^{c,2}+\delta g_R^{c,3}. \tag{27}$$

Plugging Eqs.(15)-(27) into Eq.(13), we can obtain the relative correction $\frac{\delta R_b^{LG}}{R_b^{SM}}$ given by the new gauge bosons. In our calculation, we have taken $R_b^{LG}=R_b^{SM}+\delta R_b^{LG}$, $R_b^{SM}=0.21572$, and $R_b^{exp}=0.21664\pm0.00065$[16]. Our numerical results are summarized in Fig.1 and Fig.2, in which we have used the horizontal solid line to denote the central value of R_b^{exp} and dotted lines to show the 1σ and 2σ bounds. In Fig.1(Fig.2) we plot R_b^{LG} as a function of the mixing parameter $c(c')$ for $c'=\frac{1}{\sqrt{2}}(c=\frac{1}{\sqrt{2}})$ and the scale parameter $f=1TeV$(solid line), $2TeV$(dashed line), $3TeV$(dotted line) and $4TeV$(dotted-dashed line). From Fig.1 and Fig.2, one can see that the contributions of the new gauge bosons to R_b decrease as the scale parameter f increasing. For $c'=\frac{1}{\sqrt{2}}$, R_b^{LG} is insensitive to

the mixing parameter c and the value of δR_b^{LG} is very small. For $c = \frac{1}{\sqrt{2}}$, the new gauge bosons decrease the value of the branching ratio R_b for $c' > 0.72$ and $f > 1TeV$. To make the predicted R_b value satisfy the precision experimental value in 2σ bound, we should have $0.57 < c' < 0.73$ for $f = 1TeV$. For $c = \frac{1}{\sqrt{2}}$, $f > 2TeV$, the predicted value of R_b is consistent with the precision experimental value R_b^{exp} within 2σ bound in most of the allowed range of the mixing parameter c'.

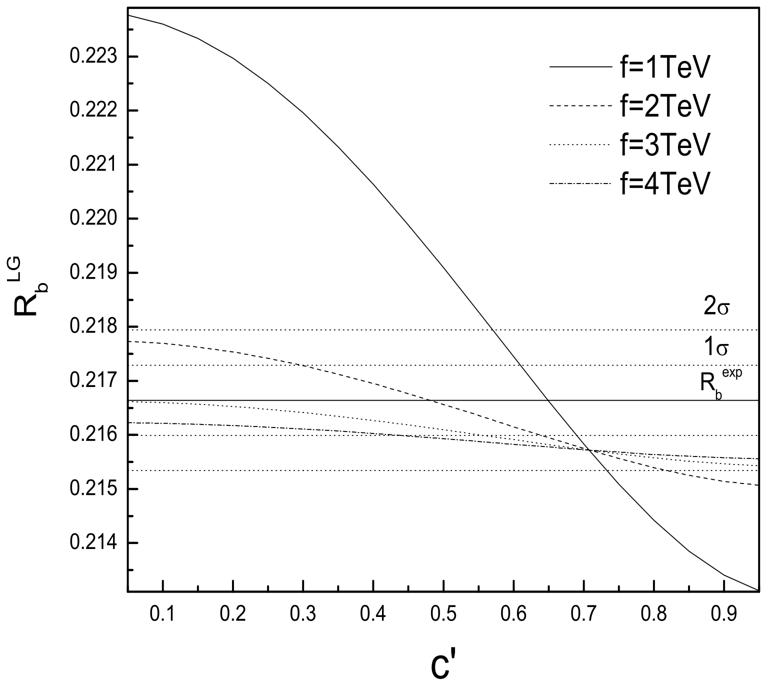

Figure 2: The branching ratio R_b^{LG} as a function of the mixing parameter c' for $c = \frac{1}{\sqrt{2}}$ and $f = 1TeV$, $2TeV$, $3TeV$ and $4TeV$.

From above discussions, we can see that the corrections of new gauge bosons to R_b can be divided into two parts: one part is the tree-level corrections coming from the shift in the Z couplings to quarks and the modifications of the relations between the SM parameters and the precision electroweak input parameters and the second part is the one-loop corrections coming from the neutral gauge bosons Z' exchange and B' exchange. To compare the tree-level corrections with the one-loop corrections, we plot $R = |\delta R_b^{1-loop}/\delta R_b^{tree-level}|$ as a function of the mixing parameter c' for $f = 2TeV$ and $c = \frac{1}{\sqrt{2}}$ in Fig.3. One can

see from Fig.3 that the one-loop contribution is smaller than the tree-level contribution at least by two orders of magnitude in all of the parameter space.

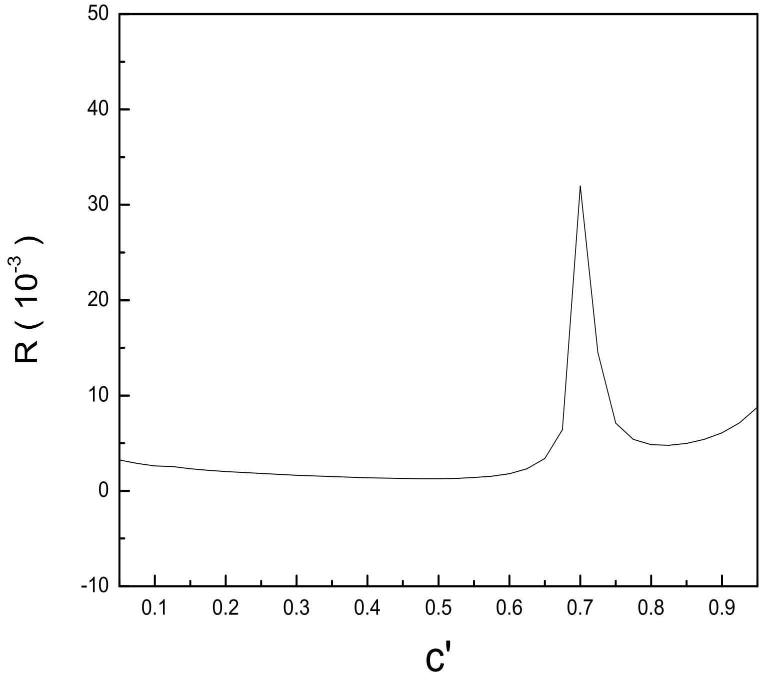

Figure 3: The relative correction R as a function of the mixing parameter c' for $f = 2TeV$ and $c = \frac{1}{\sqrt{2}}$.

IV. The corrections of the top quark t and vector-like quark T to R_b

It is well known that R_b is almost independent of the EW oblique and QCD corrections. However, for $Zb\bar{b}$ couplings, there is an important correction coming from the top triangle diagrams, which can not be ignored. They can generate significant m_t-enhanced contributions to R_b[17]. Furthermore, the extra top quark predicted by NP can also produce significant corrections to R_b at one-loop[1,18]. In this section, we will calculate the corrections of the top quark t and vector-like quark T to R_b via the couplings $W\bar{t}b$, $W\overline{T}b$, $W'\bar{t}b$ and $W\overline{T}b$. The relevant Feynman diagrams are shown in Fig.4.

Since the gauge bosons W and W' can only couple to the left handed quark s, t, T and b, the top and vector-like top triangle loops have no contributions to the right-handed

$Zb\bar{b}$ coupling g_R^b. If we assume that the mass of the bottom quark is approximately equal to zero, then the corrections to the $Zb\bar{b}$ coupling g_L^b generated by the $W\bar{t}b$ and $W'\bar{t}b$ couplings can be written as:

$$\delta g_{Lt}^{b,1} = (\frac{e}{S_W C_W})\{-\frac{\alpha}{4\pi S_W^2}[F_1(x_t) + \frac{c^2}{s^2}F_1(x_t')] + \frac{3\alpha C_W^2}{8\pi S_W^2}[F_2(x_t) + \frac{c^2}{s^2}F_2(x_t')]\} \tag{28}$$

with

$$F_1(x) = \frac{g_L^t}{2}[\frac{x(x-2)}{(x-1)^2}\ln x + \frac{x}{x-1}] + g_R^t[\frac{x}{(x-1)^2}\ln x - \frac{x}{x-1}], \tag{29}$$

$$F_2(x) = \frac{x^2}{(x-1)^2}\ln x - \frac{x}{x-1}, \tag{30}$$

where $x_t = (\frac{m_t}{m_W})^2$ and $x_t' = (\frac{m_t}{M_{W'}})^2$. In above equation, we have neglected the interference effects between gauge bosons W' and W.

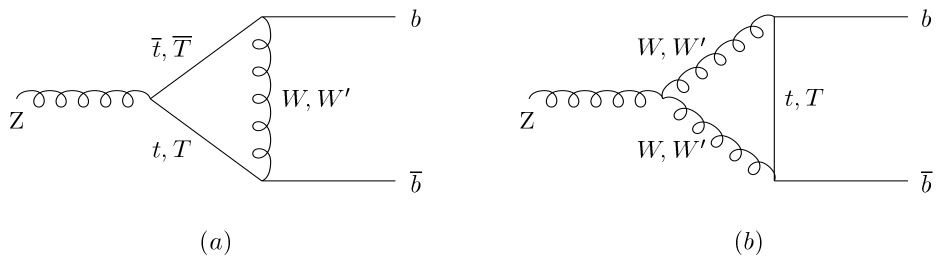

Figure 4: Feynman diagrams for the contributions of the top quark t and vector-like quark T to the $Zb\bar{b}$ vertex .

In the LH model, due to the mixing between the top quark t and the vector like quark T, the tree-level $Zt\bar{t}$ couplings receive corrections at the order of $\frac{\nu^2}{f^2}$, which also have contributions to the $Zb\bar{b}$ coupling g_L^b:

$$\delta g_{Lt}^{b,2} = -(\frac{e}{S_W C_W})\frac{\alpha}{16\pi S_W^2}(\frac{\nu^2 x_L^2}{f^2})[x_t(2 - \frac{4}{x_t - 1}\log x_t) + \frac{c^2}{s^2}x_t'(2 - \frac{4}{x_t' - 1}\log x_t')]. \tag{31}$$

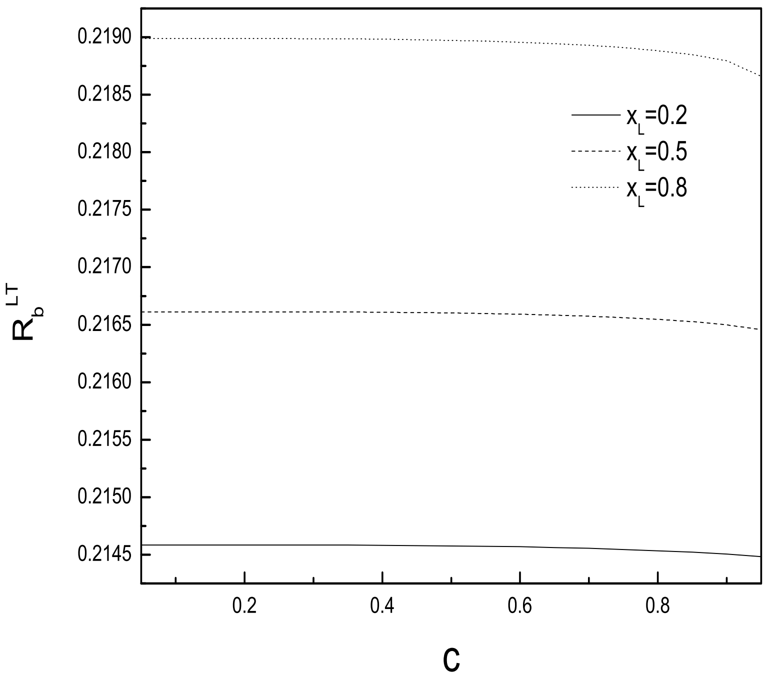

Figure 5: The branching ratio R_b^{LT} as a function of the mixing parameter c for $f = 2TeV$ and three values of the scale parameter x_L.

The mixing angle parameter between the SM top quark t and the vector-like quark T is defined as $x_L = \frac{\lambda_1^2}{\lambda_1^2+\lambda_2^2}$, in which λ_1 and λ_2 are the coupling parameters.

The contributions of the vector like top quark T to R_b via the couplings $W\overline{T}b$ and $W'\overline{T}b$ can be written as:

$$\delta g_{Lt}^{b,3} = (\frac{e}{S_W C_W})\frac{\nu^2 x_L^2}{f^2}\{-\frac{\alpha}{4\pi S_W^2}[F_3(x_T) + \frac{c^2}{s^2}F_3(x_T')] + \frac{3\alpha C_W^2}{8\pi S_W^2}[F_2(x_T) + \frac{c^2}{s^2}F_2(x_T')]\} \quad (32)$$

with

$$F_3(x) = \frac{g_L^T}{2}[\frac{x(x-2)}{(x-1)^2}\ln x + \frac{x}{x-1}] + g_R^T[\frac{x}{(x-1)^2}\ln x - \frac{x}{x-1}], \quad (33)$$

where $x_T = (\frac{M_T}{m_W})^2$ and $x_T' = (\frac{M_T}{M_{W'}})^2$. The $t - T$ contributions can be given by

$$\begin{aligned}\delta g_{Lt}^{b,4} \;=\; & (\frac{e}{S_W C_W})\frac{\alpha}{4\pi S_W^2}(\frac{\nu x_L}{f})[\frac{1}{x_T - x_t}(\frac{x_T^2}{x_T-1}\log x_T - \frac{x_t^2}{x_t-1}\log x_t) \\ & -\frac{x_t x_T}{x_T - x_t}(\frac{x_T}{x_T-1}\log x_T - \frac{x_t}{x_t-1}\log x_t)]. \end{aligned} \quad (34)$$

Being CKM suppression, the contributions of the top quark t and vector-like quark T to the couplings of the gauge boson Z to other quarks(u, c, d, s) are very small, which can be ignored. Then Eq.(13) should be changed as, for calculating the contributions of the quarks t and T:

$$\delta R_b = 2R_b^{SM}\{\frac{g_L^b \delta g_L^b + g_R^b \delta g_R^b}{(g_L^b)^2 + (g_R^b)^2} - \frac{g_L^b \delta g_L^b + g_R^b \delta g_R^b}{2[(g_L^c)^2 + (g_R^c)^2] + 3[(g_L^b)^2 + (g_R^b)^2]}\}. \tag{35}$$

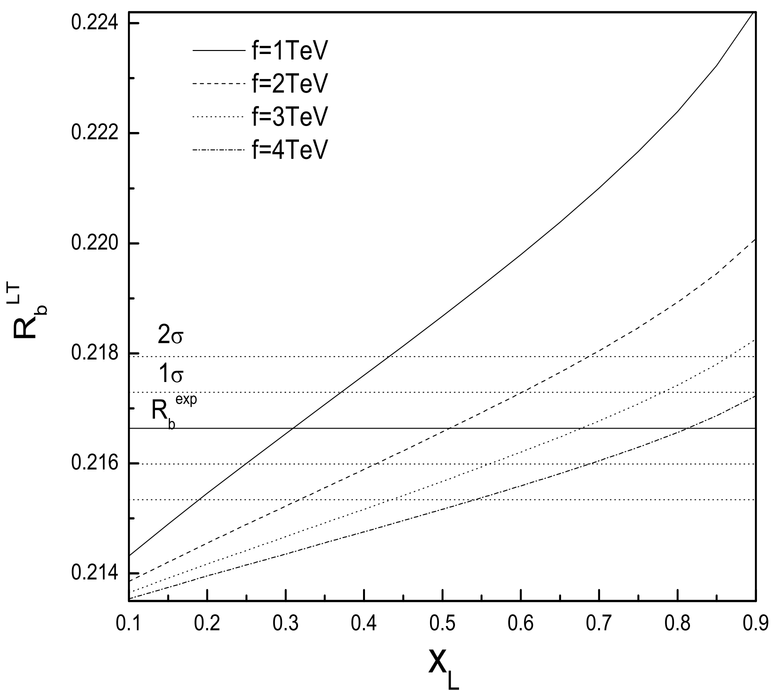

Figure 6: The branching ratio R_b^{LT} as a function of the mixing parameter x_L for $c = \frac{1}{\sqrt{2}}$ and four values of the scale parameter f.

In Fig.5 we plot the branching ratio $R_b^{LT} = R_b^{SM} + \delta R_b^t + \delta R_b^T$ as a function of the mixing parameter c for $f = 2TeV$ and three values of the mixing parameter x_L. One can see from Fig.5 that the corrections of the top quark t and vector-like quark T to R_b are not sensitive to the value of the flavor mixing parameter c, while are strongly dependent on the mixing parameter x_L. This is because the mass $M_{W'}$ of the heavy gauge boson W' suppresses the contributions of the t and T quarks to R_b. To see the effects of x_L varying on R_b, we plot R_b^{LT} as a function of x_L for $c = \frac{1}{\sqrt{2}}$, and four values of the scale

parameter f in Fig.6. One can see from Fig.6 that the top quark t and vector-like quark T can generate negative corrections to R_b for $f \geq 1TeV$ and $x_L \leq 0.25$, while they can give positive corrections to R_b for $f \leq 4TeV$ and $x_L \geq 0.66$.

V. The contributions of scalars to the branching ratio R_b

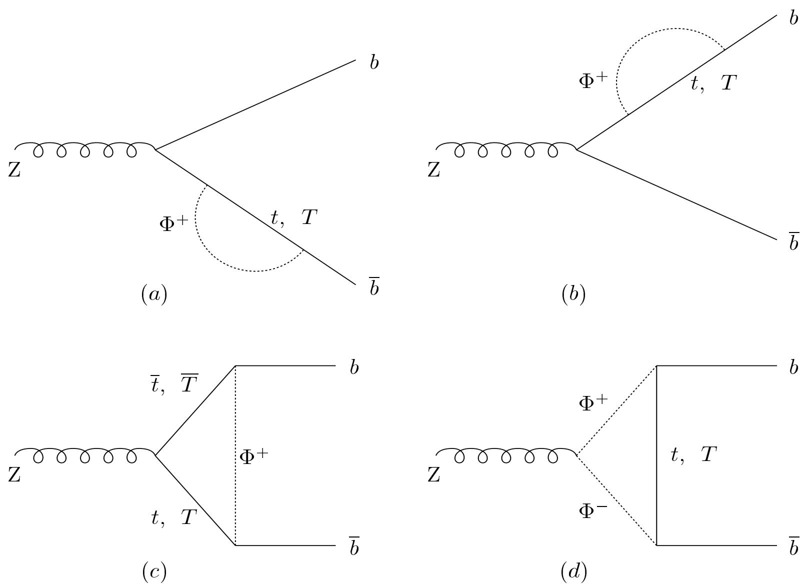

Figure 7: Feynman diagrams for the contributions of the charged scalars $\Phi^{\pm}$ to the $Zb\bar{b}$ vertex via the couplings $\Phi\bar{t}b$ and $\Phi\overline{T}b$.

Since the doubly charged scalars can not couple to the SM fermions, so they have no contributions to R_b. The singly charged scalars $\Phi^{\pm}$ can give contributions to the branching ratio R_b via the couplings $\Phi\bar{t}b$ and $\Phi\overline{T}b$. The relevant Feynman diagrams for the corrections of the charged scalars $\Phi^{\pm}$ to the $Zb\bar{b}$ couplings g_L^b and g_R^b are shown in Fig.7.

Using the Feynman rules given in appendix C and other Feynman rules, we can give:

$$\begin{aligned} \delta g_R^{b,s} &= 0, \\ \delta g_L^{b,s} &= \frac{e}{S_W C_W}\frac{m_t^2}{32\pi^2\nu^2}(\frac{\nu}{f} - 4\frac{\nu'}{\nu})[A_t + \frac{x_L}{1-x_L}A_T], \end{aligned} \qquad (36)$$

with

$$\begin{aligned} A_q &= -g_L^{b,SM}\overline{B_1}(-P_b, m_q, M_\Phi) + g_R^{t,SM}[2\overline{C}_{24}^*(P_b, -k, M_\Phi, m_q, m_q) + \overline{B_0}(-k, m_f, m_q) \\ &\quad -M_\Phi^2 C_0^*(P_b, -k, M_\Phi, m_q, m_q)] + m_t^2 g_L^{t,SM} C_0^*(P_b, -k, M_\Phi, m_q, m_q) \\ &\quad +s_W^2 \overline{C}_{24}(-P_b, k, m_q, M_\Phi, M_\Phi), \end{aligned} \tag{37}$$

where q represents the SM top quark t or the vector-like quark T. B_i, C_i and C_{ij} are the standard Feynman integrals[19], in which the variable P_b is the momentum of b quark, k is the momentum of the gauge boson Z.

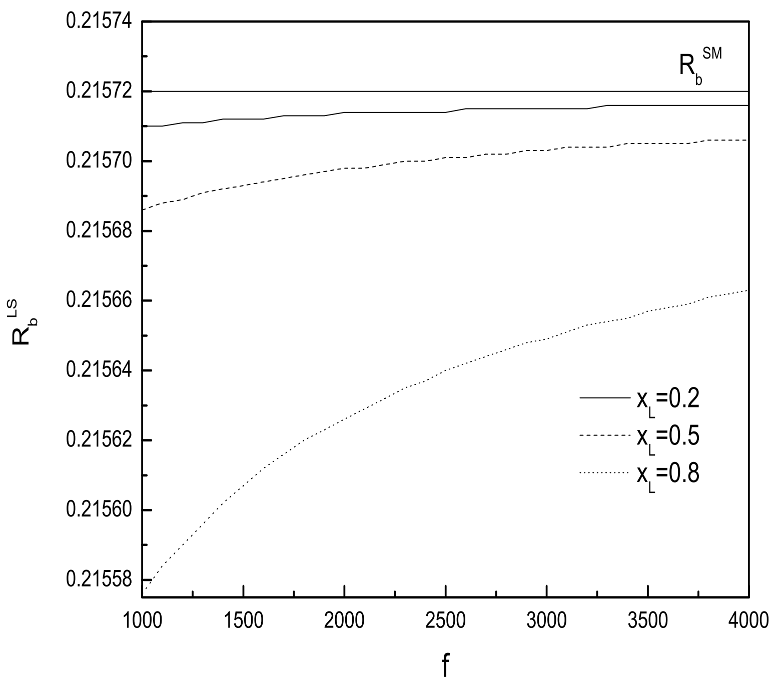

Figure 8: The branching ratio R_b^{LS} as a function of the scale parameter f for three values of the mixing parameter x_L.

Certainly, the neutral scalars H^0 and Φ^0 can also generate corrections to the $Zb\bar{b}$ couplings g_L^b and g_R^b via the couplings $H^0 b\bar{b}$ and $\Phi^0 b\bar{b}$. However, compared with the charged scalar contributions, the neutral scalar contributions are suppressed at least by the factor $\frac{m_b^2}{m_t^2}$ and thus can be ignored. In order to get a positive definite mass M_Φ of the triplet scalars, we should have that the value of the ratio of the scalar triplet VEV

ν' to the scalar doublet VEV ν is smaller than $\frac{\nu}{4f}$. To simply our calculation, we assume $\frac{\nu'}{\nu} = \frac{1}{5}\frac{\nu}{f}$. In this case, the triplet scalar mass M_Φ given in appendix A can be written as: $M_\Phi = 10m_H^2 f^2/\nu^2$, which m_H is the SM Higg mass.

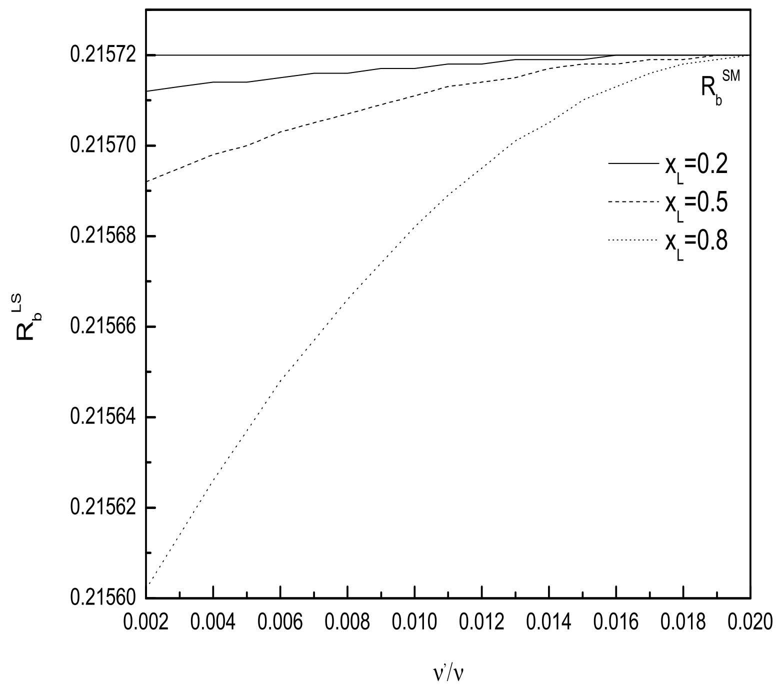

Figure 9: The branching ratio R_b^{LS} as a function of $\frac{\nu'}{\nu}$ for $f = 3TeV$ and $x_L = 0.2, 0.5$ and 0.8.

In Fig.8 we plot the branching ratio $R_b^{LS} = R_b^{SM} + \delta R_b^{LS}$ as a function of the scale parameter f for $x_L = 0.2$(solid line), 0.5(dashed line), and 0.8(dotted line), in which we have taken the SM Higgs mass $m_H = 120GeV$. We can see from Fig.8 that the charged scalars $\Phi^\pm$ generate the negative corrections to the branching ratio R_b. The negative corrections increase as the scale parameter f decreasing and the mixing parameter x_L increasing. For the parameter $f \to \infty$, the corrections of the charged scalars to R_b go to zero. However, the varying value of R_b is very small and is smaller than that generated by the new gauge bosons, the top quark t and vector-like quark T in most of the parameter space.

To see the effects of varying the triplet scalar VEV ν' on the branching ratio R_b, we

take $f = 3TeV$, which means $\frac{\nu'}{\nu} < \frac{\nu}{4f} = 0.0205$. The R_b^{LS} is plotted in Fig.9 as a function of $\frac{\nu'}{\nu}$ for $f = 3TeV$ and three values of the mixing parameter x_L. From Fig.9 we can see that the contributions of the charged scalars to R_b decrease as the value of the ratio $\frac{\nu'}{\nu}$ increasing for the fixed value of the parameter f and $\frac{\nu'}{\nu} < \frac{\nu}{4f}$. If we assume that the value of the ratio $\frac{\nu'}{\nu}$ goes to $\frac{\nu}{4f}$, then the correction value goes to zero.

VI. Discussions and conclusions

The LH model predicts the existence of several scalars, new gauge bosons, and vector-like quark T. These new particles can generate corrections to the branching ratio R_b. Thus, the predicted value of R_b can be written as $R_b^{LH} = R_b^{SM} + \delta R_b^{LG} + \delta R_b^{LT} + \delta R_b^{LS}$ in the LH model. So, using the experimental value R_b^{exp}, we might give the constraints on the free parameters of the LH model.

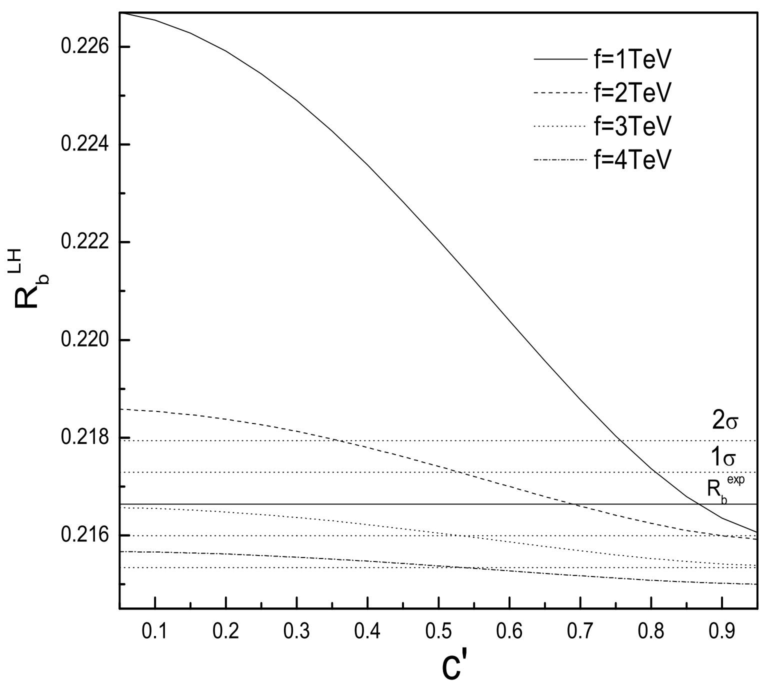

Figure 10: The predicted value of R_b^{LH} in the LH model as a function of the mixing parameter c' for four values of the scale parameter f.

From above discussions one can see that the correction effects of the new particles

predicted by the LH model to the branching ratio R_b decrease as the scale parameter f increasing. The charged scalars generate the negative correction to R_b in all of the parameter space. The correction value increases as the mixing parameter x_L increasing, which is very small. The contributions of the top quark t and the vector-like T are related to the parameters c and x_L. However, they are insensitive to the parameter c, while are strongly dependent on the parameters x_L and f. The new gauge bosons, such as Z' and B', can give corrections to R_b at tree-level and one-loop. The one-loop contributions are smaller than the tree-level contributions at least by two orders of magnitude in most of the parameter space. These contributions are sensitive to the parameters c' and f. Thus, the total correction of the LH model to the branching ratio R_b is mainly dependent on the parameters f, c' and x_L. Thus, we can take the parameters c and $\frac{\nu'}{\nu}$ as fixed value: $c = \frac{1}{\sqrt{2}}$ and $\frac{\nu'}{\nu} = \frac{\nu}{5f}$ for calculating the total correction to R_b.

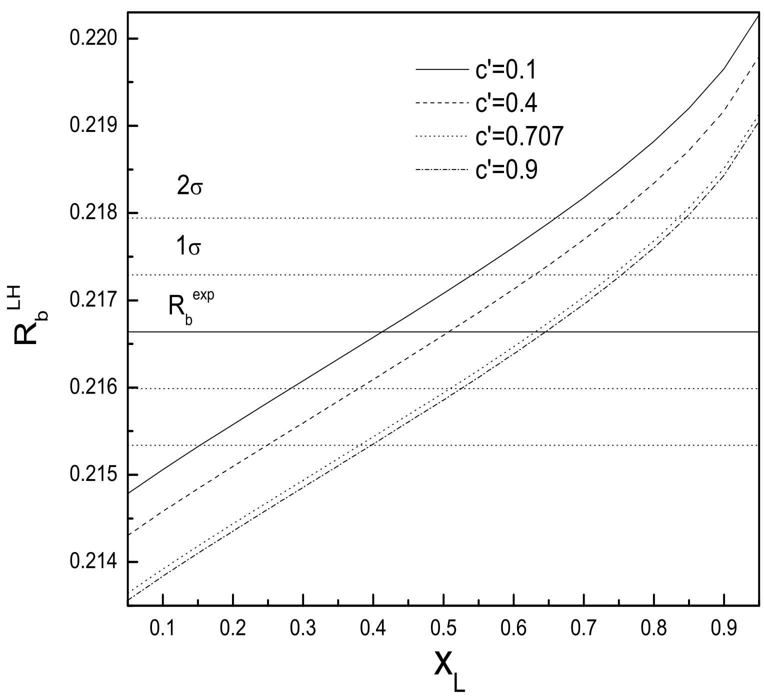

Figure 11: The predicted value of R_b^{LH} in the LH model as a function of the mixing parameter x_L for four values of the mixing parameter c'.

In Fig.10 we plot the branching ratio R_b^{LH} as a function of the mixing parameter c'

for $x_L = 0.5$ and four values of the scale parameter f. From Fig.10 we can see that the value of R_b^{LH} decreases as the parameter c' increasing. For $f = 1TeV$, the value of R_b is too large to consistent with the precision experimental value R_b^{exp} in most of the parameter space. Furthermore, the large value of the scale parameter f is in favor of the general expectation based on other phenomenological explorations. Thus, in Fig.11, we take $f = 3TeV$ and plot the R_b^{LH} as a function of the mixing parameter x_L for four values of the parameter c'. From Fig.11 we can see that the value of R_b^{LH} decreases as the parameter x_L increasing. If we demand that the predicted value R_b^{LH} consistent with the precision experimental value R_b^{exp} within 2σ bound for $f = 3TeV$, there must be:

$$c' = 0.1, \qquad 0.16 \leq x_L \leq 0.67; \qquad c' = 0.4, \qquad 0.25 \leq x_L \leq 0.74;$$
$$c' = \frac{1}{\sqrt{2}}, \qquad 0.38 \leq x_L \leq 0.84; \qquad c' = 0.9, \qquad 0.39 \leq x_L \leq 0.83.$$

If we take the small value for the scale parameter f, these constraints will became more strong. For example, for $f = 2TeV$ and $c' = c = \frac{1}{\sqrt{2}}$, we have $0.28 \leq x_L \leq 0.65$ in order to R_b^{LH} consistent with R_b^{exp} within 2σ bound.

Little Higgs models have generated much interest as possible alternatives to weak scale supersymmetry. The LH model is a minimal model of this type, which realizes the little Higgs idea. In this paper, we study the corrections of the new particles predicted by the LH model to the branching ratio R_b. We find that the corrections of the neutral scalars to R_b is very small, which can be neglected. The charged scalars can generated the negative corrections to R_b. The new gauge bosons and fermions might generate the positive or negative corrections to R_b, which dependent on the values of the mixing parameters c, c' and x_L. If we demand that the contributions of the new gauge bosons and fermions cancel those generated by the charged scalars and make the predicted value R_b^{LH} consistent with the precision experimental value R_b^{exp}, then the parameters x_L, c' and f must be severe constrained.

Acknowledgments

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Appendix A: The masses of the gauge bosons Z' and B', triplet scalar Φ, and the vector-like quark T.

The masses of the gauge bosons Z', B' and W' can be written at the order of $\frac{\nu^2}{f^2}$:

$$
\begin{aligned}
M_{Z'}^2 &= m_Z^2 C_W^2 [\frac{f^2}{s^2c^2\nu^2} - 1 - \frac{5S_W^3}{2C_W} \cdot \frac{sc(c^2s'^2 + s^2c'^2)}{s'c'(5C_W^2 s'^2c'^2 - S_W^2 s^2c^2)}], && (38)\\
M_{B'}^2 &= m_Z^2 S_W^2 [\frac{f^2}{5s'^2c'^2\nu^2} - 1 + \frac{5C_W^3}{8S_W} \cdot \frac{s'c'(c^2s'^2 + s^2c'^2)}{sc(5C_W^2 s'^2c'^2 - S_W^2 s^2c^2)}], && (39)\\
M_{W'}^2 &= m_Z^2 c_W^2 (\frac{f^2}{s^2c^2\nu^2} - 1), && (40)
\end{aligned}
$$

where m_Z is the mass of the SM gauge boson Z, ν is the electroweak scale.

For the triplet scalar Φ, we have

$$
M_\Phi^2 = 2m_H^2 \frac{f^2}{\nu^2} \frac{1}{1 - (\frac{4f}{\nu}\frac{\nu'}{\nu})^2}, \tag{41}
$$

where m_H is the SM Higgs mass and ν' is the triplet scalar vacuum expectation value(VEV).

The mass of the heavy vector-like quark T can be written as:

$$
M_T = \frac{m_t f}{\nu} \sqrt{\frac{1}{x_L(1-x_L)}} [1 - \frac{\nu^2}{2f^2} x_L (1 + x_L)], \tag{42}
$$

where m_t is the SM top quark mass, x_L is the mixing parameter between the SM top quark and the heavy vector-like quark T, which is defined as $x_L = \frac{\lambda_1^2}{\lambda_1^2+\lambda_2^2}$. λ_1 and λ_2 are the Yukawa coupling parameters.

Appendix B: The relevant coupling constants of the gauge bosons to fermions.

$$
\begin{aligned}
W\bar{t}b : g_L^{\bar{t}b} &= \frac{ie}{\sqrt{2}S_W}[1 - \frac{\nu^2}{2f^2}(x_L^2 + c^2(c^2 - s^2))]V_{tb}^{SM}, && (43)\\
g_R^{\bar{t}b} &= 0, && (44)
\end{aligned}
$$

where V_{tb}^{SM} is the SM CKM matrix element. In our calculation, we will take $V_{tb}^{SM} = 1$.

$$
\begin{aligned}
W\overline{T}b: &\quad g_L^{\overline{T}b} = \frac{e}{\sqrt{2}S_W}\frac{\nu}{f}x_L V_{tb}^{SM}, \qquad g_R^{\overline{T}b} = 0. && (45)\\
W'\bar{t}b: &\quad g_L^{\bar{t}b} = -\frac{e}{\sqrt{2}S_W}\frac{c}{s}V_{tb}^{SM}, \qquad g_R^{\bar{t}b} = 0. && (46)
\end{aligned}
$$

$$W'\overline{T}b: \quad g_L^{\bar{t}b} = -\frac{e}{\sqrt{2}S_W}\frac{\nu}{f}x_L\frac{c}{s}V_{tb}^{SM}, \qquad g_R^{\bar{t}b} = 0. \tag{47}$$

$$Zb\bar{b}: \quad g_L^b = \frac{e}{S_W C_W}\{-\frac{1}{2}+\frac{1}{3}S_W^2+\frac{\nu^2}{f^2}[\frac{c^2(c^2-s^2)}{4} + \frac{5}{6}(c'^2-s'^2)(\frac{1}{5}-\frac{1}{2}c'^2)]\}, \tag{48}$$

$$g_R^b = \frac{e}{S_W C_W}[\frac{1}{3}S_W^2+\frac{5}{3}\frac{\nu^2}{f^2}(c'^2-s'^2)(\frac{1}{5}-\frac{1}{2}c'^2)].$$

$$Zt\bar{t}: \quad g_L^t = \frac{e}{S_W C_W}\{1-\frac{2}{3}S_W^2-\frac{\nu^2}{f^2}[x_L^2+\frac{c^2(c^2-s^2)}{4} + \frac{5}{2}(c'^2-s'^2)(\frac{4}{5}-c'^2+\frac{2}{3}s'^2x_L)]\}, \tag{49}$$

$$g_R^t = \frac{e}{S_W C_W}\{-\frac{2}{3}S_W^2-\frac{\nu^2}{f^2}5(c'^2-s'^2)[\frac{3}{5}-c'^2(1-\frac{1}{3}x_L+\frac{2}{15}x_L)]\}.$$

$$ZT\overline{T}: \quad g_L^T \approx g_R^T = \frac{e}{S_W C_W}(-\frac{2}{3}S_W^2), \tag{50}$$

$$Zt\overline{T}: \quad g_L^{tT} = -i\frac{e}{S_W C_W}\frac{x_L\nu}{4f}, \qquad g_R^{tT} = 0. \tag{51}$$

$$ZW^+W^-: \quad g^{ZWW} = g^{ZW'W'} = -\frac{eC_W}{S_W} \tag{52}$$

$$Z'b\bar{b}: \quad g_L^b = -\frac{e}{2S_W}\cdot\frac{c}{s}, \qquad g_R^b = 0 \tag{53}$$

$$Z't\bar{t}: \quad g_L^t = \frac{e}{S_W}\cdot\frac{c}{2s}, \qquad g_R^t = 0. \tag{54}$$

$$Z'T\overline{T}: \quad g_L^T = g_R^T = -\frac{e}{S_W C_W}(\frac{2}{3}S_W^2). \tag{55}$$

$$B'b\bar{b}: \quad g_L^b = \frac{e}{3C_W s'c'}(\frac{1}{5}-\frac{1}{2}c'^2), \qquad g_R^b = \frac{2e}{3C_W s'c'}(-\frac{1}{5}+\frac{1}{2}c'^2). \tag{56}$$

$$B'c\bar{c}: \quad g_L^c = \frac{e}{3C_W s'c'}(\frac{1}{5}-\frac{1}{2}c'^2), \qquad g_R^c = \frac{4e}{3C_W s'c'}(\frac{1}{5}-\frac{1}{2}c'^2). \tag{57}$$

Appendix C: The coupling constants of the scalars to fermions.

$$H^0b\bar{b}: \quad -i\frac{m_b}{\nu}(1-4\frac{\nu'^2}{\nu^2}+2\frac{\nu'}{f}-\frac{2}{3}\frac{\nu^2}{f^2}). \tag{58}$$

$$\Phi^0b\bar{b}: \quad -i\frac{m_b}{\sqrt{2}\nu}(\frac{\nu}{f}-4\frac{\nu'}{\nu}). \tag{59}$$

$$\Phi^Pb\bar{b}: \quad \frac{m_b}{\sqrt{2}\nu}(\frac{\nu}{f}-4\frac{\nu'}{\nu}). \tag{60}$$

$$\Phi^+\bar{t}b: \quad -\frac{i}{\sqrt{2}\nu}[m_tP_L+m_bP_R](\frac{\nu}{f}-4\frac{\nu'}{\nu}), \tag{61}$$

with $P_L = \frac{1-\gamma_5}{2}, \qquad P_R = \frac{1+\gamma_5}{2}$.

$$\Phi^+\overline{T}b: \qquad -i\frac{m_t}{\sqrt{2}\nu}(\frac{\nu}{f} - 4\frac{\nu'}{\nu})\sqrt{\frac{x_L}{1-x_L}}P_L. \tag{62}$$

The coupling vertex of the SM gauge boson Z to the charged scalars $\Phi^{\pm}$ is

$$i\frac{e}{S_W C_W}S_W^2(P_1 - P_2)_\mu.$$

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